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Identifying present bias and time preferences with an application to land-lease-contract data [1]

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Summary: What can contracts—traded and priced in a competitive market and featuring a pre-specified system of future payments—teach us about time preferences and present bias? We first show that identification of present bias requires assumptions on the felicity function and that agents must have credit constraints on consumption expenditure. Moreover, when there is heterogeneity in present bias, identification requires that agents with the same present bias parameter buy houses with different contracts. We illustrate our findings with observational land-lease-contract data from Amsterdam.

Keywords: *Structural modelling*, *present bias*, *identification*.

JEL codes: *C10*, *D10*, *O18*.

1. INTRODUCTION

In many cities (including Amsterdam, London, and Singapore), a substantial fraction of houses are leaseholds, which are temporary and tradeable ownership rights of (a part of) the property. The lease is typically prepaid for up to 50–100 years and in some cases even extends to 1000 years.[2] This paper first explores whether data on land-lease contracts can be used to investigate the existence of present bias. More ambitiously, we ask under what conditions this type of data can be used to identify the short- and long-run discount factor. It turns out that the conditions for the identification of these estimands are strong . We suggest an estimation method and illustrate its use with simulations and actual land-lease data from Amsterdam, where we compare houses on private land with leaseholds.

In order to illustrate how data on leasehold contracts can be informative on present bias, we turn to a simple example. Suppose that we observe two identical houses (I and II) that are sold at

[1] An older version of this paper has circulated since 2011 under the title: 'A flexible test for present bias and time preferences using land-lease contracts'. We thank John Rust and two anonymous referees for excellent comments.

[2] Giglio et al. (2015) use these leaseholds to estimate the prices for different maturities. Examples of other papers that use comparable data include Wong et al. (2008), Tyvimaa et al. (2015), Fesselmeyer et al. (2016), and Gautier and van Vuuren (2019).

a particular moment in time.[3] Suppose, further, that house I is located on privately owned land, while the owner of house II must pay 2000 euros in land-lease rent per year, starting ten years from now. In a competitive housing market, the selling price of house I will obviously be higher. What assumptions must be made in order to use this price difference (and that of other identical houses with different land-lease contracts) to identify the short- and long-run discount factors? Some form of credit constraint will be required; otherwise, buyers will be able to neutralize any difference in the timing of the payments between the two houses by either borrowing or saving against the market interest rate.[4] Luttmer and Mariotti (2003) show that if consumers can borrow up to the present value of their endowments, and utility growth is constant (or when utility functions are logarithmic), agents will consume a constant part of their wealth irrespective of the shape of the subjective discount function. This implies that equilibrium prices are the same as when consumers discount geometrically, even when they do not, in fact, do so.

Since it is almost impossible to accumulate large amounts of debt when one does not own a house (which can be used as collateral), it is reasonable to assume that agents can borrow to finance a house but that they face credit constraints on consumption (although they can save at the market interest rate).[5] In terms of the above example, the agent who buys the house with land lease faces lower mortgage payments today but must pay more after ten years at the start of a new land-lease term. Therefore, this agent has more income available for consumption in the first ten years, and less afterwards. If consumers smooth their consumption over time (in response to predictable income shocks), then present bias affects the way they do this—which demonstrates how consumption data can be informative on time-preference parameters.[6]

When consumers are credit-constrained, there will be a direct relationship between land-lease payments and consumption—and this will affect the price of houses. One complication is that a house where the land-lease payments start immediately and an identical freehold both generate a constant stream of payments (mortgage cost plus land lease) and hence a constant consumption flow over time. Therefore, only the contracts that are prepaid for a substantial amount of time (but substantially less than the time horizon) can give consumers flexibility in their consumption pattern (because of the changes in housing costs over time). Agents without present bias find that such contracts are valuable because they allow them to relax their credit constraint. The lower the long-run discount factor, the more valuable this is for them, and consequently the greater is their willingness to pay for houses with those contracts. Agents with present bias value those contracts quite differently. When they are sophisticated, they realize that their future selves will behave in a time-inconsistent way, so they have a desire now to buy houses with contracts that allow them to commit to a constant consumption pattern. Houses without land lease and houses with non-prepaid land lease offer them this opportunity and are thus perceived as extra valuable. If houses with those contracts are relatively expensive, then this suggests that (some) agents have present bias. We formalize this by devising a dynamic programming model with credit constraints and present bias that is closely related to the model in Harris and Laibson (2001).

[3] The data contain many observable characteristics that jointly explain 95% of the variance in house prices. Thus, in the empirical part of the paper, we can control for almost all of the differences in house characteristics.

[4] Similar points were made by Coller and Williams (1999), Harrison et al. (2002), and Frederick et al. (2002).

[5] Those assumptions are almost identical to assuming that it is possible to borrow without collateral but against a much higher rate than the savings and mortgage interest rate. Moreover, credit card debt is low in the Netherlands. About half of the population above the age of 18 own a credit card; the average card debt is only about 200 Euros per card.

[6] One could also compare prices of houses where the land lease has been prepaid for one period with houses that are not prepaid; we choose, however, not to rely on that because identification then relies too much on what exactly a period length is in the $\beta-\delta$ quasi-hyperbolic discounting model.

Unfortunately, identification requires parametric assumptions on the consumers' felicity function. When this function is linear, there is no need to smooth consumption; house prices will then only be affected by changes in the long-run discount factor.[7] If the aim is to investigate whether there is present bias or not, we show that an advantage of using the Constant Absolute Risk Aversion (CARA) felicity function is that it is the one where it is hardest to detect present bias. Consequently, for this case it is sufficient to show that the present bias parameter is less than one.

We also show that when agents are heterogeneous in terms of their present bias parameter (β), we need variation in contract types among agents with the same β. As in the assignment and hedonics literature,[8] when there is perfect sorting of buyer types into different contracts, the model is not identified. Therefore, the number of different contracts must be large relative to the number of buyer types. Otherwise, it is possible that heterogeneous buyers (who all discount geometrically) generate the same data as identical buyers that have present bias. A comparable problem exists in the market for durable goods (such as cars), where consumers differ in terms of their taste for the physical condition of the good; see Rust (1985).[9]

On top of that, identification of the short- and long-run discount factors requires assumptions on initial wealth, switching costs and the term structure of interest rates.[10] We make those assumptions explicit and show which assumptions can be relaxed when there is more data available than merely house prices and land-lease contracts.

We present simulations illustrating how model misspecification affects the estimation results. Our simulation exercise suggests that model misspecification has little effect on our long-run discount factor (δ) estimates, but that if we erroneously assume CARA and homogeneity, while in fact agents are heterogeneous in their present bias and have Constant Relative Risk Aversion (CRRA) utility, the true level of β is overestimated.

In a reduced-form companion paper (Gautier and van Vuuren, 2019), we provide direct evidence that houses with lease contracts that have been prepaid in advance for many years sell at a higher price than identical houses that have been prepaid for fewer years. Postponed payments are thus priced in.[11] However, as we show in our identification results, this by itself does not imply that there is present bias.

Our structural-form estimates suggest a short-run discount factor ($\beta\delta$) of 0.82 and a long-run discount factor (δ) equal to 0.96, corresponding to a 22.5% short-run and a 4.2% long-run discount rate. If we allow for heterogeneity, this means that 61% of the population has a substantially lower short-run discount factor (i.e., $\beta\delta = 0.74$). Even though we reject the null of no present bias, the improvement in the goodness of fit is modest.

The short- and long-run discount factors we find are a bit lower than those found in Laibson et al. (2007). Those authors identify these discount factors by matching moments on wealth accumulation, credit card borrowing and excess sensitivity of consumption to predictable movements in income.[12]

[7] See also Andersen et al. (2008), Andreoni and Sprenger (2012a,b), and Harrison et al. (2002).

[8] See, for example, Tinbergen (1956), Rosen (1974), and Heckman et al. (2010).

[9] Gillingham et al. (2019) solve this problem by using multiple transactions of individual car buyers. Even though not impossible, such multiple transactions are harder to find for real-estate transactions.

[10] van Binsbergen et al. (2015) use dividend strips to estimate the term structure of the equity premium. Our framework does not take this into account. It is possible that agents with present bias will also choose a different term structure, and this can potentially bias our results.

[11] We find that houses on private land are on average 13% more expensive than houses with land lease. Each additional year of prepayment makes a house 0.2% more expensive.

[12] They find a short-term discount factor equal to 0.90 and a long-term discount factor of 0.96, implying a short-term discount rate of 15.8% and a long-term discount rate of 3.8%.

Most of the evidence for present bias comes from lab experiments; see Frederick et al. (2002) for an overview. We believe that evidence from the field can provide important additional information. As noted by Laibson et al. (2007) and Della Vigna (2017), many studies have estimated general discount functions with lab data, but only a few papers use observational data to estimate the degree of present bias, and none of those studies discusses identification with observational contract data.[13] Housing market data are potentially interesting because the stakes are high, implying that the incentives to invest in information and to calculate (or let an expert calculate) the pay-offs over time are large as well. Of course, the general drawbacks of using field data (in terms of lack of control of the environment) remain, which means that we must make additional identifying assumptions.

Several papers have used leasehold data to estimate the long-run discount rate. Giglio et al. (2015), for example, find for the UK and Singapore that leaseholds trade with a discount even in the case that the termination date is up to 700 years in the future. In line with our results, this indicates that the long-run discount rate is very low. Warner and Pleeter (2001) use field data from the U.S. military drawdown program to estimate discount rates. This program offered separatees the choice between an annuity and a substantial lump-sum payment (the stakes are thus also high in their setting). Most of the separatees selected the lump sum, implying a discount rate that exceeds 17%. An advantage of their data is the availability of individual characteristics. Cho and Rust (2017), investigating the credit card transactions of a large Korean bank, give evidence for pre-commitment that is difficult to explain using standard models of utility maximization.[14]

The paper is organized as follows. Section 2 presents a model of consumption and housing. Section 3 discusses identification. Section 4 discusses how the model can be estimated and how model misspecification will affect the estimates. Section 5 presents an empirical illustration using land-lease data. Section 6 concludes.

2. A MODEL OF CONSUMPTION AND HOUSING

We consider a discrete-time model where agents can spend their income and wealth on consumption and housing. We distinguish between M different agent types who differ in how they discount future consumption. A fraction χ_m of the population has type m. Let H be a vector of housing attributes, and $c_{s,m}$ be the number of units of the composite consumption good that an agent of type m buys in period s. Apart from the mortgage payments, home-buyers have to pay an amount equal to L_s^j in period s, where $\mathcal{L}_s^j = \{L_t^j; t = s+1, \ldots\}$ is the full sequence of these payments. In our empirical illustration, L_s^j will be land-lease payments that are specified in a contract. We refer to these contracts as land-lease contracts, but the results can be applied to any type of tradeable contract that specifies payments or donations in the future. Subscript j indicates that the different future payment schemes correspond to different contracts. We assume that there are J different contracts and denote the fraction of houses with a contract j by μ_j. Define $p(H, \mathcal{L}_s^j)$ to be the price of a house with characteristics H in case this house is sold with

[13] Examples of studies that use observational data include Ahumada and Garegnani (2007), Attanasio and Weber (1995), Attanasio et al. (1999), Shui and Ausubel (2004), Paserman (2008), Fang and Silverman (2009), and Kaur et al. (2015). With regard to issues surrounding identification of present bias using monetary rewards, see for example the discussions in Augenblick et al. (2015), Cubitt and Read (2007), and Chabris et al. (2008). Abbring et al. (2019) discuss exclusion restrictions on the utility function that allows identification of present bias in dynamic discrete choice models.

[14] Relatively high prices of houses with land-lease contracts that offer buyers the chance to pre-commitment to a low consumption level in the present are not only consistent with present bias but also with models of self-control; see for example Thaler and Shefrin (1981).

a land-lease contract $\mathcal{L}_s^j$. Finally, agents have income y and an amount $\omega_{s,m}$ of liquid assets in period s. Note that neither the mortgage nor the value of the house is included in the assets. We make the following assumptions, which will be discussed below:

ASSUMPTION 2.1 (UTILITY FUNCTION) *Utility is additively separable in housing and in the composite consumption good with felicity functions $u_H(\cdot)$ and $u_C(\cdot)$ for housing and consumption.*

ASSUMPTION 2.2 (DISCOUNT FUNCTION) *Agents have a quasi-hyperbolic discount function: i.e., current consumption is not discounted and the discount function for agent type m is given by $\beta_m\delta^s$, where $1 \geq \beta_1 \geq \beta_2 \geq ... \geq \beta_m$ and $\delta \leq 1$.*

ASSUMPTION 2.3 (SOPHISTICATION) *Agents are sophisticated (they know today that they have present bias in the future).*

ASSUMPTION 2.4 (HOUSE CHARACTERISTICS) *The land-lease contract $\mathcal{L}_s^j$ is not an element in the vector H.*

ASSUMPTION 2.5 (TIME HORIZON) *The time horizon, T, is finite.*

ASSUMPTION 2.6 (BUDGET RESTRICTION) *Agents can only borrow for housing services and not for consumption, $\omega_{s,m} \geq 0$, $s = 0, \ldots, T$.*

ASSUMPTION 2.7 (INCOME) *Income is deterministic.*

ASSUMPTION 2.8 (INITIAL WEALTH) *Initial wealth equals zero: i.e., $\omega_{0,\,m} = 0$.*

ASSUMPTION 2.9 (SUBSTANTIAL SWITCHING COSTS) *After buying a house, agents do not move.*

ASSUMPTION 2.10 (NO TERM STRUCTURE) *The interest rate equals r and is assumed to be both the lending and savings rate. It does not depend on the term.*

ASSUMPTION 2.11 (TIMING OF COSTS) *The first payments (both mortgage and land lease) are at the end of the first period after the house is bought.*

ASSUMPTION 2.12 (SUFFICIENT DISCOUNTING) $\beta_m\delta < 1/(1+r); m = 1, ..., M$.

Assumption 2.1 is made for convenience, making it possible for us to focus on the composite consumption good. The functional form in Assumption 2.2 is the quasi-hyperbolic discount function of Laibson (1997) with heterogeneity in the short-run discount rate. Assumption 2.3 is necessary because we have to take a stance on whether agents are naive or sophisticated (see also the discussion in Della Vigna, 2017). Although sophistication complicates the analysis, we think that for important decisions like buying a house, agents either make careful decisions or hire an expert to assist them in doing so. Assumption 2.4 implies that, conditional on the timing and size of the payments, the buyer receives no additional utility from a particular payment scheme. Assumption 2.5 allows us to use backward induction, which results in a unique optimal consumption path. Assumption 2.6 is essential to identify our discount function and can be motivated by the fact that houses can be used as collateral, whereas other consumption goods cannot.[15,16] Assumptions 2.7 and 2.8 are driven by data restrictions. We have very detailed housing data but no data on the wealth or income of the home owners. Section 3.4 discusses how these assumptions can be relaxed when one observes wealth and income data. The role of Assumption 2.9 is to rule out that agents change houses every year to re-optimize $\mathcal{L}_s^j$. A similar issue arose in Rust (1985) for the second-hand market of durable goods (such as cars) based on the condition of that durable good. That paper did allow owners to switch when that was optimal. However, the transaction and moving costs for home buyers and sellers are in general substantially higher than the benefits of a better land-lease contract, so in our setting this assumption is more reasonable. Assumption 2.9 can be relaxed to allow agents to change house for reasons unrelated to the land-lease contract, such as a new job or changes in family composition. This would, however, make the model more complex in dimensions that are not relevant for the main question at hand in this paper. Assumption 2.10 is made for tractability. Allowing for both different term structures and present bias will make the model (and the corresponding identification problem) too complex. The role of Assumption 2.11 is to rule out the possibility that our identification results depend solely on the payments made in the present. This would also make the model dependent on the length of the period, which would be arbitrary and undesirable. Finally, Assumption 2.12 is necessary because otherwise the credit constraints are not binding (while these constraints are essential for identification).

How much the agent can consume depends on the amount of liquid assets she owns (i.e., $\omega_{s,m}$). By Assumption 2.6, the restriction $c_{s,m} \le \omega_{s,m}$; $s = 1, \ldots, T$ must hold. The law of motion for the liquid assets equals

$$\omega_{s,m} = (\omega_{s-1,m} + y - c_{s,m} - L_s^j - rp(H, \mathcal{L}_0^j))(1 + r). \tag{2.1}$$

Without loss of generality, we assume that individuals take a full mortgage on their house; the interest outlays are then equal to $rp(H, \mathcal{L}_0^j)$.[17]

Define $c_{s,m}(\omega_{s,m})$, $s = 0, \ldots, T$ as the optimal consumption path of a buyer, given that her liquid assets are equal to $\omega_{s,m}$. Let $U_{s,m}(H, \omega_{s,m}, \mathcal{L}_s^j)$ be her remaining lifetime utility when she owns a house with attributes H, has liquid assets equal to $\omega_{s,m}$ and a land-lease contract $\mathcal{L}_s^j$, and follows the optimal consumption path. She maximizes utility by choosing present consumption and housing subject to her dynamic budget constraint. Since she is sophisticated, she can also predict her future consumption path. Her utility is given by[18]

$$U_{s,m}(H, \omega_{s,m}, \mathcal{L}_s^j) = u_H(H) + \beta_m \sum_{k=s+1}^{T} u_H(H)\,\delta^{k-s} + W_{s,m}(H, \omega_{s,m}, \mathcal{L}_s^j), \tag{2.2}$$

where $W_{s,m}(H, \omega_{s,m}, \mathcal{L}_s^j)$ is defined as

$$W_{s,m}(H, \omega_{s,m}, \mathcal{L}_s^j) \equiv u_C\left(c_{s,m}(\omega_{s,m})\right) + \beta_m \sum_{k=s+1}^{T} u_C\left(c_{k,m}(\omega_{k,m})\right)\delta^{k-s}. \tag{2.3}$$

[15] Alternatively, one could assume that buyers face different lending rates for housing and consumption. Such a model would be equivalent to our case whenever the short-run discount rate is less than the lending rate for consumption. The short-run discount rate that we find in our empirical application is indeed lower than the typical interest rate that is paid on credit cards.

[16] Note that these additional assumptions solve the identification problem discussed by Luttmer and Mariotti (2003).

[17] The generality follows from the assumption that borrowing and lending rates are identical.

[18] Note that the buyers' optimization problem also includes the choice of H. We can abstract from this problem using our assumption of separability.

Note that $W_{s,m}$ depends on H only with regard to the selling price (through our dynamic budget restriction (2.1)). Because we focus on identical houses in the remainder of this section, we drop H for ease of notation and write $W_{s,m}(H, \omega_{s,m}, \mathcal{L}_s^j) = W_{s,m}(\omega_{s,m}, \mathcal{L}_s^j)$ and $p(H, \mathcal{L}_0^j) = p(\mathcal{L}_0^j)$.[19] $W_{s,m}(\omega_{s,m}, \mathcal{L}_s^j)$ depends on the land-lease contract, which in turn will affect the price of the house $p(\mathcal{L}_0^j)$. In order to indicate this dependence, we consider j different house types and write $W_{s,m}(\omega_{s,m}(p(\mathcal{L}_0^j)), \mathcal{L}_s^j)$, or, with a little abuse of notation, $W_{s,m}(p(\mathcal{L}_0^j), \omega_{s,m}, \mathcal{L}_s^j)$.

In equilibrium, agents of the same type should have the same utility level for the houses that they buy and a (weakly) lower utility for the houses that they do not buy. Define the market utility of agents of type m by $\overline{W}^m$. In order to determine the equilibrium prices and how agent types sort into different land-lease contracts, we use the concept of a bid price, which originates from the urban economics literature (Fujita, 1989). Let $\Psi_{\sigma,m}(\mathcal{L}_0^j)$ be the bid price that an agent of type m is maximally willing to pay for a house where the land lease is paid σ years in advance in order to obtain her market utility level, $\overline{W}^m$. This is implicitly defined by

$$W_0(\Psi_{\sigma,m}(\mathcal{L}_0^j), \omega_{0,m}, \mathcal{L}_0^j) = \overline{W}^m. \tag{2.4}$$

Note that the bid price $\Psi_{\sigma,m}(\mathcal{L}_0^j)$ is also defined for houses that are not bought by agents of type m. We obtain the following *price function*:

$$p(\mathcal{L}_0^j) = \arg\max_q \left\{ q \,\middle|\, \sum_{m=1}^{M} \chi_m \mathbf{1}(\Psi_{\sigma,m}(\mathcal{L}_0^j) \geq q) \geq \mu_i \right\}, \tag{2.5}$$

where the right-hand side gives the set of prices, q, for which demand for type j houses is at least as high as supply. We assume that the equilibrium price equals the highest of these prices, so sellers receive the seller-optimal point in the core.[20] In equilibrium, we also have that

$$\sum_{j=1}^{J} \mathbf{1}\left\{\Psi_{\sigma,m}(\mathcal{L}_0^j) \geq p(\mathcal{L}_0^j)\right\} \mu_j \geq \chi_m. \tag{2.6}$$

This accounting condition states that the measure of houses that agents of type m are willing to buy is weakly larger than the measure of agents m. Note that we have an inequality because we have a discrete set of contract and agent types.

Buyers maximize lifetime utility from consumption based on (2.3) and subject to the dynamic budget constraint (2.1), $c_{s,m} \geq 0$, and the transversality condition, i.e., $\omega_{T,m} = 0$. This is a dynamic programming model with credit constraints and present bias. In order to solve it, we draw heavily on the heuristic derivations presented in Harris and Laibson (2001). Our model is more tractable than theirs owing to Assumptions 2.5 and 2.7.[21] For illustrative purposes, we consider only those buyers who bought a house that has no additional payments for the next σ years, and from now onwards, we simply denote the dependence of the house price on σ by p_σ. Moreover, we assume that there is no variation in L_s^j other than the jump at $s = \sigma$. This implies that $L_s^j = 0$ whenever $s \leq \sigma$, and that $L_s^j \equiv L$ afterwards. Because contracts depend on σ only, we denote them by $\mathcal{L}_s^\sigma$. Let $V_{s,m}$ be the continuation value of the buyer (i.e., the future value evaluated today, so we

[19] We deal with the case of heterogeneous houses in our empirical application.

[20] Note that this assumption is not essential, since we look at relative rather than at absolute prices.

[21] Thanks to these assumptions, it can be shown that the main problem of the heuristic model of Harris and Laibson (2001) (i.e., that the consumption function is non-differentiable in the consumer's initial wealth) does not exist for our case.

discount just with δ), which equals

$$V_{s,m}(p_\sigma, \omega_{s,m}, \mathcal{L}_s^\sigma) = u_C\left(c_{s,m}(\omega_{s,m})\right) + \delta V_{s+1,m}(\omega_{s+1,m}, \mathcal{L}_{s+1}^\sigma). \tag{2.7}$$

In line with (2.2), let $W_{s,m}$ be the current value (so now we discount at rate $\beta_m\delta$) for a buyer with a land-lease contract,

$$W_{s,m}(p_\sigma, \omega_{s,m}, \mathcal{L}_s^\sigma) = u_C\left(c_{s,m}(\omega_{s,m})\right) + \beta_m\delta V_{s+1,m}(\omega_{s+1}, \mathcal{L}_{s+1}^\sigma). \tag{2.8}$$

Moreover, $c_{s,m}(\omega_{s,m})$ is determined by the following constrained maximization problem:

$$c_{s,m}(\omega_s) = \underset{c\in[0,\omega_{s,m}]}{\arg\max}\left\{u_C(c) + \beta_m\delta V_{s+1,m}(\omega_{s+1,m}, \mathcal{L}_{s+1}^\sigma)\right\}, \tag{2.9}$$

where $\omega_{s,m}$ is defined in (2.1). Appendix A shows that under certain (empirically verifiable) restrictions of the discount rates, the optimal consumption path weakly decreases and equals

$$c_{s,m}(\omega_{s,m}) = \begin{cases} y - rp_\sigma & s \le s_m^* \\ C_{s,m}(\omega_{s,m}) & s_m^* < s \le s_m^{**} \\ y - L - rp_\sigma & s > s_m^{**} \end{cases}, \tag{2.10}$$

where the consumption level $C_{s,m}(\omega_{s,m})$ is implicitly determined by

$$\log\left[u_C'\left(C_{s,m}(\omega_{s,m})\right)\right] = \log[u_C'(c_{s_m^*,m}(\omega_{s_m^*,m}))] - \left(s - s_m^*\right)\log((1+r)\beta_m\delta) - \sum_{k=0}^{s-s_m^*-1}\log\left(c_{s-k,m}'(\omega_{s-k,m}) + \frac{1}{\beta_m}\left(1 - c_{s-k,m}'(\omega_{s-k,m})\right)\right), \tag{2.11}$$

and where s_m^* and s_m^{**} are the boundary points between which the buyer is smoothing her consumption (before and after those points, all available income is consumed).[22] Figure 1 illustrates the boundary points s_m^* and s_m^{**}. The solid line is income available for consumption. There is a drop in income at σ due to the fact that the buyer has to pay for land lease from that period onwards. The dotted line represents consumption. Up to s_m^*, the buyer prefers to consume more than her income available for consumption, $y - rp_\sigma$. Therefore, the credit constraint is binding. At s_m^*, her preferred consumption has fallen below her income available for consumption, and she starts to save for future consumption and hence accumulates wealth. From σ onwards, she consumes those assets up to s_m^{**}, when all assets are depleted. In this region, her unrestricted desired consumption is below her income available for consumption. On her unrestricted consumption path, the low level of consumption was due to the fact that she had to pay back the loans on past consumption. In the restricted case, since she cannot borrow, she consumes all her available income from s_m^{**} until the end of the time horizon.

The way in which consumption is smoothed between s_m^* and s_m^{**} depends on the shape of the felicity function (and hence on the willingness of consumers to intertemporally substitute their consumption). Define $A(c) = -u_C''(c)/u_C'(c)$. It equals the absolute value of the elasticity of marginal felicity divided by c.[23] In the special case that $\beta = 1$, this also equals the reciprocal of the elasticity of intertemporal substitution (see Laibson, 1996). The following proposition is proved in Appendix B.

[22] The formal definitions of s^* and s^{**} are in the working paper version of our paper.

[23] $A(c)$ equals the coefficient of absolute risk aversion, but we decided to omit this terminology because our model does not include any income uncertainty.

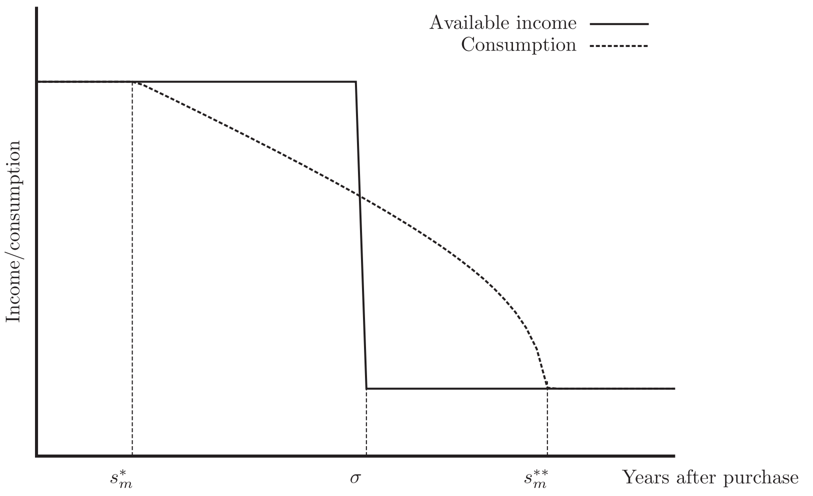

Figure 1. Illustration of the boundary points.

PROPOSITION 2.1 *Define* $\Delta_m(s) = |C_{s+1,m} - C_{s,m}|$.

(1) When $\beta_m = 1$, *then* $\Delta_m(s) > (<)\Delta_m(s-1)$ *whenever* $A(c) > (<)A(c-\Delta)$ *for any* c, $c, \Delta > 0$.

(2) When $\beta_m = 1$ *and* $A(c) \geq A(c-\Delta)$, *then* $\Delta_m(s) > \Delta_m(s-1)$.

Proposition 2.1 states that if $\beta_m = 1$, then the drop in income available for consumption results in a concave relationship of the consumption function over time for the case that $A(c)$ is increasing in consumption (whereas we obtain a convex relationship for the case that $A(c)$ is decreasing in consumption). Consumption decreases linearly whenever $A(c)$ is constant. The second, less-known, implication of Proposition 2.1 states that under hyperbolic discounting the only way we can obtain a convex relationship of consumption over time is by using a felicity function $A(c)$ that is decreasing in consumption. A special case is the CRRA felicity function.

Figure 2 illustrates the impact of present bias on consumption for the case that $A(c)$ is constant with respect to c. Again, the solid line represents available income. The dashed line represents consumption under present bias, while the dotted line represents the case without present bias where the short- and long-run discount rates are the same. By Proposition 2.1, the case without present bias implies a linear evolution of consumption over time. In contrast, under present bias, consumption decreases rapidly just before s_m^{**}. In addition, individuals with present bias smooth their income for a shorter period of time.

3. IDENTIFICATION

3.1. Sketch of the identification problem

To understand the potential problems of identifying β and δ in our setup, we first introduce a simple three-period model with homogeneous agents and then extend the model to heterogeneous agents. When there are three periods, there are four types of contracts (i.e., land-lease payments

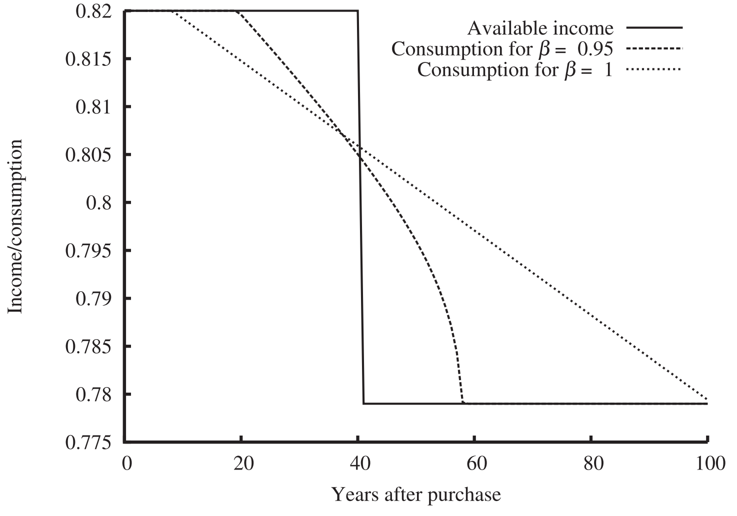

Figure 2. Consumption patterns for different levels of the short-term discount factor (β).

that do not have to be made for 0, 1, 2 or all 3 years). The corresponding contracts are called, respectively, type 0, 1, 2 and 3 contracts. Denote the fraction of houses of type σ by μ_σ. When the utility of consumption is linear in income and $\beta_m = 1$, then we only have to consider the discounted mortgage and land-lease payments of a house. For houses with type 0 contracts, these payments are equal to:

$$rp_0(\delta + \delta^2 + \delta^3) + L(\delta + \delta^2 + \delta^3).$$

For houses with type 1, 2 and 3 contracts, the first term is similar, except that p_0 is replaced by p_1, p_2 and p_3, respectively. The second term describes the discounted lease payments. Those are equal to $L(\delta^2 + \delta^3)$ for type 1 contracts, to $L\delta^3$ for type 2 contracts, and to 0 for type 3 contracts.

Since agents and houses are identical, the discounted consumption stream accompanying each contract type should have the same value (contracts with a lower value would not be traded). In other words, the discounted costs for all houses should be equal to each other. Equating the discounted costs of houses with type 0 and 3 contracts yields

$$rp_3 = rp_0 + L \Leftrightarrow p_3 = p_0 + \frac{L}{r}. \tag{3.1}$$

From this equation, it follows that price variation in houses with type 0 or type 3 contracts does not help in the identification of any of the model parameters. This is because the first type is associated with higher payments for land lease and the second type with a higher mortgage, but the total monthly costs are the same and constant over time. In general,

$$p_\sigma = p_{\sigma-t} + \frac{L}{r}\frac{\Sigma_{s=1}^{\sigma-t}\delta^s}{1+\delta+\delta^2}, \tag{3.2}$$

which implies that $\partial(p_1 - p_0)/\partial\delta < 0$, while $\partial(p_2 - p_1)/\partial\delta > 0$ and $\partial(p_3 - p_2)/\partial\delta > 0$. Hence, the first-order derivative is non-monotonic. When $\delta < 1$, agents discount future consumption and would like to borrow in order to increase current consumption. Credit constraints prevent them from doing this. However, agents can relax their credit constraint by buying houses with contracts where they pay relatively little in the beginning, i.e., type 1 or type 2 contracts. Unlike the houses with type 0 or type 3 contracts, the total payments for these houses are low in the first period(s) and high in the remaining periods. For example, for type 1 contracts, the payment is equal to $rp_1 < rp_3$ in the first period, while in the second and third periods, it is equal to $rp_1 + L > rp_0 + L$. This allows the buyers of houses with type 1 contracts to consume relatively more in the first periods and less in later periods. This opportunity is more valuable the more impatient the agent is. The fact that the relationship is non-monotonic follows directly from the fact that there is a fixed difference between p_0 and p_3; see the discussion following (3.1). Hence, if a decrease in δ increases the difference between p_1 and p_0, then it should also decrease the difference between p_3 and p_1. Note that this non-monotonicity implies that a simple linear regression of the price on the number of years paid in advance is not informative on the level of δ.

Next, we relax the assumptions that the felicity function is linear and that $\beta_m = 1$ for all groups. The identification results do not change much when we allow for nonlinear felicity functions if $\beta_m = 1$. However, there is a big difference when $\beta_m = \beta < 1$. In that case, our identifying equations based on the discounted costs for type $\sigma \in \{0, 1, 2\}$ contracts can be written as

$$rp_\sigma(\delta + \delta^2 + \delta^3)\beta + L \sum_{s=\sigma}^{2} \delta^{s+1}\beta.$$

For type 3 contracts, the L expression drops out. In equilibrium, the discounted costs for all contracts must again be identical. However, since by Assumption 2.11 all payments take place in the future, β drops out of the equilibrium equations and we obtain the same equations as in (3.2). Therefore, β is not identified when the felicity function is linear. When the felicity function is nonlinear and concave, consumption responds differently to changes in β and δ, since in that case, agents have a desire to smooth their income over time. This results in two counteracting forces. On the one hand, impatient agents (i.e., low-δ agents) prefer to consume more now and less later, implying that buying a house for which there are some prepaid periods is beneficial by the mechanism described above. On the other hand, agents prefer to smooth their consumption over time. Consider, for example, the lifetime budget restriction of an agent who buys a house with land lease that is paid one period in advance. It equals

$$(y - rp_1 - c_1)(1 + r)^2 + (y - rp_1 - L - c_2)(1 + r) + y - rp_1 - L - c_3 = 0.$$

Agents who buy such a house can perfectly smooth their consumption pattern by choosing

$$c = y - rp_1 - \frac{L(2 + r)}{2 + r + (1 + r)^2}.$$

This does not need to be the optimal consumption pattern, but we can use it to calculate the value of commitment by comparing the prices that agents are willing to pay when using this consumption pattern with the prices that agents are willing to pay if they can freely choose their consumption pattern. When $\beta = 1$, buyers are always willing to pay more for a house with a type 1 contract without constraints on consumption than for one with a type 1 contract with the constraint that they must consume the same amount each period. This result is independent of the utility function. The difference in value depends on δ and the intertemporal rate of substitution.

This changes dramatically for the case that $\beta < 1$. This is based on two observations. First, we already know that a lower level of β does not imply that an agent prefers a house with

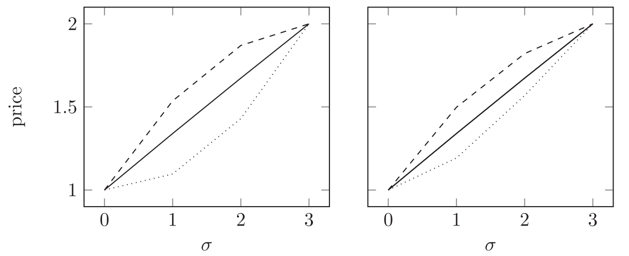

Figure 3. Contract prices for different levels of β and δ.
Notes: The dashed lines represent the case with $\beta = 1$ and $\delta < 1$, and the dotted lines represent the case with $\delta = 1$ and $\beta < 1$. In the right-hand panel, the credit constraints are binding for $\sigma = 2$, while in the left-hand panel they are not.

variable payments. This is because all benefits and costs of any contract are in the future, and hence have no impact on the lifetime utility of the agents (other than the fact that δ might be less than one). Second, if $\beta < 1$, agents can use contracts that come with a fixed consumption pattern as a commitment. That is, sophisticated agents prefer to constrain themselves to a smooth consumption pattern because this prevents time-inconsistent behaviour. Compared with agents without present bias, they are willing to pay less for contracts that are paid only one or two periods in advance.

Figure 3 shows the discussed relationships for the case of a CARA felicity function. The left-hand panel is the simplest case, where we assume that the credit constraints are not binding. The solid line plots the case where $\delta = \beta = 1$ and where consumption is constant over time (for the CARA case, it is a straight line). The dashed curve is for the case where $\delta < 1$ and $\beta = 1$. As discussed above, when δ is low, type 1 and type 2 contracts are relatively valuable. The dotted line represents the case where $\delta = 1$ and $\beta < 1$. When β is low, contracts that imply a varying consumption pattern over time (i.e., type 1 and type 2 contracts) are less valuable than contracts that allow the agents to commit to a fixed consumption pattern (type 0 and type 3 contracts). When $\beta < 1$ and $\delta = 1$, agents are willing to pay less for contracts that allow for more consumption in the beginning and less in the future (when both δ and β are less than one, this is, however, not necessarily the case).

Thus far, we have abstracted from the question of whether or not the agents' credit constraints are binding. Identification requires that they are binding in some periods but not in all. If they are always binding, agents consume all of their income in every period, and then, as in the linear felicity-function case, β is not identified. When the credit constraints are not binding, β and δ cannot be jointly identified (see also Luttmer and Mariotti, 2003). This can be seen in the left-hand panel of Figure 3 for type 1 and type 2 contracts, where for the chosen parameters, the credit constraints are only binding in periods zero and three. Identification fails because in the area between $\sigma = 1$ and $\sigma = 2$ the dotted and dashed lines are parallel to each other. Hence, a linear combination of β and δ can explain the price differences between type 1 and type 2 contracts. The right-hand panel of Figure 3 plots the case where the credit constraints are binding only for type 2 contracts. In that case, the impact of β on the price is zero for type 2 contracts while it is positive for type 1 contracts. This implies that β and δ can be jointly identified. In practice, we observe many contract types and different values of land-lease rent, so there will be enough identifying information in the data (there will always be some contract types for which the credit constraint is binding and others for which it is not).

Next, we introduce buyer heterogeneity. Suppose that there are two types of agents, namely one type with no present bias ($\beta = 1$) and the other with present bias ($\beta < 1$). Let χ be the fraction of individuals with present bias. Now, the prices become bid prices, which differ from market prices because they are observed only for the agent type with the lowest valuation who buys a particular contract. For agents who do not buy a house with a particular contract, we only know that such a house generated less value than the contract(s) that this type did buy. In addition, since in equilibrium agents must be indifferent between the types of houses that they are willing to buy, the difference in prices between two identical houses with different contracts that are bought by a particular agent type must be equal to the difference in bid prices of that agent type. This implies that in order to identify the discount factor of a given agent type i, we must observe that type i agents buy houses with different contracts. Moreover, these contracts cannot be only type 0 and type 3 contracts because (as we showed before) these imply the same consumption pattern and provide no independent identifying information on either β or δ.

Note that agents without present bias cannot be the only buyers of houses with type 0 or type 3 contracts. This is because those houses come with a flat consumption pattern, and agents without present bias strongly prefer to pay less in the beginning. Only type 1 and type 2 contracts allow for this. So if some agents without present bias sort into type 0 or type 3 contracts, agents with present bias (who prefer constant consumption patterns and use those contracts as a commitment device) will also do that.

If there are enough agents without present bias in relation to type 1 and type 2 contracts (i.e., $\chi < 1 - \mu_1$ and $\chi < 1 - \mu_2$), then it is possible to identify δ by using the prices of the houses of type 1 and type 2 contracts, which are then equal to the bid rents of the agents with $\beta = 1$. That is, we have at least one equation (the difference in bid rents) and one unknown (i.e., the true level of δ). This does not immediately imply that β is also identified. For the identification of β, we must also have that χ is sufficiently large (i.e., $\chi > 1 - \mu_1 - \mu_2$) in order to have agents with present bias to buy houses with contracts that are prepaid for either one or two periods. Again, in that case, the price difference between contracts reflects the difference in bid rents for these time-inconsistent agents. For a given δ, this yields at least one other restriction and only one unknown value (the true value of β).

Identification becomes easier when there are more contract types than agent types or when there are search frictions, because then we are more likely to observe variation in contract types among the different agent types. This also makes the requirement that credit constraints are binding in some but not all periods more likely to hold. In terms of Figure 1, we only need that s^* and s^{**} do not equal 0 and T, respectively.

To sum up, this example shows that even when one has data on contracts that specify payments at different points in time that are traded in a competitive market, identification of time-preference parameters can only be satisfied when there is no perfect sorting of buyer types into specific contracts.

3.2. Identification in the T period model and the felicity function

The previous section demonstrated that the short-run discount factor must have a direct impact on the consumption path in order to be identified. This is a necessary condition but it is not sufficient, since uniqueness is also required. That is, there should not be two (or more) sets of

β_m and δ that result in the same consumption pattern over time. Proposition 2.1 is informative about the potential problems facing identification, since it shows that a concave relationship of consumption smoothing can be explained by a combination of a decreasing level of $A(c)$ and a high level of present bias or by no present bias and an increasing level of $A(c)$. This implies that even when we have homogeneous agents, β and δ cannot be identified as long as we do not make additional restrictions with respect to the felicity function. Fortunately, we have the following result, which is proved in Appendix C.

PROPOSITION 3.1 *If agents are homogeneous, then for any given pattern of consumption, the underlying value of β for a felicity function with $A(c + \Delta) < A(c)$ is strictly lower than for the case that $A(c + \Delta) = A(c)$.*

To understand the intuition behind Proposition 3.1, suppose first that the observed consumption pattern is convex over time. Then, assuming a constant $A(c)$ would immediately rule out any present bias—whereas a felicity function with a decreasing $A(c)$ can still be consistent with present bias. The analysis is more difficult when the consumption pattern is concave. In that case, we can prove that a lower level of β is necessary to explain such a concave relationship whenever the felicity function exhibits a decreasing $A(c)$ than when $A(c)$ is constant.

Proposition 3.1 implies that if we want a lower bound on the degree of present bias (or an upper bound on β), we should use a CARA felicity function, i.e., $u_C(c) = (1 - \exp(-ac))/a$, which is the only felicity function with a constant $A(c)$. It implies that our estimates of β can be seen as an upper bound; hence, if we find present bias under the CARA felicity function, then there must also be present bias if the true shape of the felicity function is different (e.g., of the CRRA type). The CARA felicity function reduces to a linear felicity function when $a \downarrow 0$. We also estimate the model for this limiting case.

Since we assume here that we only observe house prices, we need one additional step to show that both β and δ are estimable from housing and land-lease data. This step is relegated to Appendix D; we only provide the intuition below.

Figure 3 illustrates how house prices are decreasing in δ. This figure is based on three time periods but easily extends to the general case. Houses that do not have to pay land lease for one or two years allow for more consumption today, but this has less value when δ is higher. This implies that if we are only interested in the identification of δ, then it would suffice to know the price if the house is located on own land and the price of a type 1 or type 2 contract. However, things become more complicated if we want to identify β and δ simultaneously. Obviously, this identification cannot be performed by looking at just one price, since that would just give one restriction on two unknowns. Nevertheless, we are able to use the price information of different contracts to solve for both β and δ. However, this only yields a unique solution in the case that these contracts give restrictions that are independent. More formally, define the relative impact of β and δ on the house price $\widehat{p}$ by $h(\sigma) = (\partial p_\sigma/\partial\beta)/(\partial p_\sigma/\partial\delta)$. Then, we need to show that $h(\sigma)$ is not a constant, since otherwise all restrictions would be linearly dependent (as in the left-hand panel of Figure 3 between $\sigma = 1$ and $\sigma = 2$). We show in Appendix D that $h(\sigma)$ is not constant as long as s^* and s^{**} are (a) not equal to each other and (b) not equal to 0 and T. Then, the prices are independent of β for the periods where the credit constraints are binding, while they do depend on δ (by the timing of consumption). Hence, as long as the credit constraints are binding in some periods, β and δ affect prices differently for different levels of σ.

3.3. *Identification when multiple transactions are available*

Since we have thus far assumed that buyers cannot move, we do not observe multiple transactions of the same buyer over time. The question arises whether our identification results change if we relax this assumption but continue to assume that agents move for reasons that are orthogonal to contract type. Unfortunately, multiple transactions do not help for identification of the time-preference parameters when there is perfect sorting of buyers into contract types (as discussed in Section 3.1).[24] However, introducing tremblings or arbitrary small search frictions will rule out perfect sorting, and then the distribution of β_m can be nonparametrically identified if we observe multiple transactions per buyer.[25]

3.4. *Identification when consumption and income or wealth data are available*

We have thus far assumed that the only data observable to the researcher are house prices and land-lease contract data. A natural question is whether additional data on consumption, income and/or wealth can weaken our identifying assumptions.

According to Proposition 2.1, we are not able to identify the model parameters without parametric assumptions on the felicity function. Even without knowing the felicity function however, we can make qualitative statements on whether or not there is present bias. For example, we know that concave consumption patterns over time imply that individuals have present bias.

Equation (2.11) implies that $h_C(\sigma) = (\partial c/\partial\beta) \big/ (\partial c/\partial\delta)$ is not a constant function of σ for any strictly concave felicity function, which provides a sufficient condition for identification. This implies that when we observe consumption at the individual level over time, we no longer need to observe different contracts within an agent type. Additionally, with income and consumption data, Assumptions 2.7 and 2.8 are no longer necessary.

4. ESTIMATION AND SIMULATIONS

4.1. *Estimation*

Consider a house i with land-lease payments L_i that has been prepaid for σ_i more periods. Equations (2.5) and (2.6) allow us to calculate the virtual price of this house if it were a freehold (this virtual price will obviously always be higher). Suppose that besides the actual price, we also observe the price of an identical house that is located on private land. When our model is well specified, then the only difference between the virtual price and the price of the identical house on private land must come from measurement error (or differences in unobserved house characteristics). This enables us to estimate our parameters of interest by minimization of the mean squared error, i.e.,

$$\widehat{\vec{\beta}}, \widehat{\delta}, \widehat{a}, \widehat{\vec{\chi}} = \arg\min_{\vec{\beta},\delta,a,\vec{\chi}} \sum_i \left(\widehat{p}_i - \widehat{p}(p_{\sigma_i,\vec{\beta},\delta,a,\vec{\chi}})_i\right)^2, \tag{4.1}$$

[24] In a comparable setting for the second-hand car market, Gillingham et al. (2019) solve this by looking at both the buying and the selling age of the car. We suspect that even though selling a car based on its age is reasonable, it is much less likely that homeowners would sell their houses solely on the basis of a decline in the number of years that are prepaid.

[25] See, for example, Gautier and Teulings (2006) and Eeckhout and Kircher (2011) for similar issues in the labour market context.

where $\widehat{p}(p_{\sigma_i,\vec{\beta},\delta,a,\vec{\chi}})$ is the virtual price of house i with land lease that is prepaid for σ years, and $\widehat{p}_i$ is the price of a freehold that is otherwise identical to house i. Moreover $\vec{\beta} = (\beta_1, \ldots, \beta_M)$ and $\vec{\chi} = (\chi_1, \ldots, \chi_{M-1})$. Section 5 discusses some practical issues related to the fact that houses may depend on (un)observables.

4.2. Simulations

This section investigates the role of model misspecification. Suppose we assume, for example, CARA felicity with identical agents and use only price data, while one or more of those assumptions are violated in the underlying data-generating process (DGP) . How will that affect our estimates of δ and β? Also interesting to discover is the extent to which our estimates would improve if we had better data (such as additional consumption data). We consider the following three DGPs: (1) a model with CARA felicity and no differences in time preferences between agents; (2) a model with CRRA felicity and no differences in time preferences; (3) a model with CRRA felicity and two mass points of time preferences (one for $\beta - \delta$ preferences and another for δ preferences). All simulations are based on 500 observations and 100 iterations of the simulations.

For all of our exercises, we set δ at 0.95. In our DGPs without heterogeneity, we assume $\beta = 0.9$, while for the DGPs with heterogeneity, we assume that half of the population has $\beta = 0.9$ and the other half has $\beta = 1$.

We assume that all houses are identical in observed characteristics, but allow them to differ in terms of unobservable characteristics. In particular, the prices of houses on privately owned land are assumed to be normally distributed with a mean of 200 and a standard error of 1. In case consumption is observed, we assume that it is measured with error (with a standard error equal to 1). The distribution of the number of years that houses are prepaid is assumed to be log-normal with parameters $\log(18)$ and 0.6. We set the interest rate to 2% per year, the CARA parameter to 0.25 and the CRRA parameter to 2. The land-lease rent that must be paid is set at 2% of the price of a house on privately owned land. Finally, we set income equal to 30 (note that this does not affect the estimates when the true model has a CARA felicity function).

Panel A of Table 1 reports our results for the first four exercises. Simulation (1)—listed in the first column of Panel A of Table 1—is based on the CARA felicity function. Since in this case there is no model misspecification by construction, we estimate this model well. Interestingly, the standard error of the simulations for β are much larger than those for δ. This is because when β increases, consumption smoothing becomes more important (so it increases the price of houses that come with contracts that allow for this). An increase in δ also puts more weight on future land-lease and mortgage payments, so it also has a direct effect on prices. Simulation (2) of Table 1 lists the results using a CRRA felicity function and identical agents. We overestimate β when we use a CARA felicity function while the data are generated by a CRRA felicity function. Note, however, that this result is not completely obvious from Proposition 3.1, since now we estimate the CARA felicity function parameters and we use price rather than consumption data. The estimates of δ are close to its true value for all specifications, and the standard error is very low. Simulations (3) and (4) are also based on the assumption that the data are generated by a CRRA felicity function, but now with two groups of agents. Simulation (3) lists the results for the case where we wrongly assume identical agents and we wrongly assume a CARA felicity function in the estimated model. In that case, β is close to the average between the two groups, who have $\beta = 0.9$ and $\beta = 1$, respectively. The fourth column of Table 1 reports the results for the case

Table 1. Simulation results.

A. Only price data available				
	(1)	(2)	(3)	(4)
β	0.9014	0.9726	0.9747	0.8641
	(0.0101)	(0.0148)	(0.0143)	(0.0107)
δ	0.9500	0.9521	0.9530	0.9500
	(0.0005)	(0.0004)	(0.0002)	(0.0001)
a	0.2490	0.0371	0.0138	0.0399
	(0.0108)	(0.0061)	(0.0037)	(0.0020)
χ				0.4067
				(0.0260)
B. Consumption data available				
	(5)	(6)	(7)	(8)
β	0.9000	0.9097	0.9093	0.9038
	(0.0006)	(0.0143)	(0.0074)	(0.0089)
δ	0.9500	0.9426	0.9511	0.9476
	(0.0001)	(0.0066)	(0.0043)	(0.0022)
a	2.001	0.0859	2.370	0.0899
	(0.0184)	(0.0097)	(0.213)	(0.0063)
χ				0.5567
				(0.1332)

Notes: (1) CARA model homogeneous DGP and estimated; (2) CRRA model homogeneous DGP, homogeneous CARA model estimated; (3) heterogeneous CRRA model DGP, homogeneous CARA model estimated; (4) heterogeneous CRRA model, DGP heterogeneous CARA model estimated; (5) homogeneous CRRA model DGP, homogeneous CRRA model estimated; (6) homogeneous CRRA model DGP, homogeneous CARA model estimated; (7) heterogeneous CRRA model DGP, homogeneous CRRA model estimated; (8) heterogeneous CRRA model DGP, heterogeneous CARA model estimated. Standard errors of the simulations are between parentheses.

where we wrongly assume a CARA felicity function but correctly take into account that there are two groups of discounters. Here we find that β, in contrast to column 3, is underestimated. Finally, χ is also somewhat underestimated.

Panel B of Table 1 reports simulations similar to those in Panel A, but now we assume that income and consumption data are also available. Simulation (5) lists the results for the case with correctly specified homogeneous CRRA preferences. Again, unsurprisingly, we estimate the parameters for this model extremely well, in terms of both unbiasedness and precision. Note, however, that here we assume that we observe consumption data for all periods, which gives many more data points than the 500 different prices that we had earlier. Simulation (6) lists the estimation results for the same DGP but now under the wrong assumption of CARA preferences. The estimated parameters are still close to the true ones, but the standard errors increase. Simulations (7) and (8) in Table 1 list the results for a heterogeneous CRRA DGP. Simulation (7) reports the results for a model that correctly assumes a CRRA felicity function but wrongly assumes homogeneity. This leads to a modest overestimation of β (0.91 rather than 0.90). Simulation (8) reports the results for the case where we wrongly assume a CARA felicity function and again wrongly assume identical agents. The parameters are even in this case surprisingly close to the actual ones. Only the standard errors are larger than in the case of a homogeneous DGP.

5. EMPIRICAL ILLUSTRATION: LAND LEASE IN AMSTERDAM

5.1. *Land lease*

Land lease is defined as the right to hold and to use the land. In our application, the land is owned by the city of Amsterdam. To obtain this right, the leaseholder must pay the city an annual fee, called the 'land-lease rent'. Land lease is different from tenancy because it can be traded without the owner's intervention. The city of Amsterdam has used land-lease contracts since 1896. Before that year, all land was sold; since that year, the city of Amsterdam has always remained the owner of the land. For more details, see Gautier and Van Vuuren (2019).

One important feature of land lease is that it can be paid in advance until the end of the contract period. Prepayments are also traded.[26]

5.2. *Data*

We use data from two sources. Our first data set comes from the city of Amsterdam and contains all land-lease contracts that were effective between 2007 and 2017. It distinguishes between houses for which the land-lease rent is paid in advance and houses for which the land-lease rent must be paid yearly, starting immediately. These are all individual residential houses. For these houses we have the identifier of the house from the Dutch register, the beginning and end date of the contract, and the beginning and the end date for which the land-lease rent has been paid in advance. Besides information on the general conditions of the land-lease contract, we have information on the special conditions of the payment period and the exact amount that has to be paid annually during the years of observation.

Our second data set, from the Dutch Association of Real-Estate Agents (NVM), contains information on more than 70% of the houses that were sold in Amsterdam within our observation window. The data are from the beginning of January 1985 until the end of December 2017. A large set of characteristics is available for every house (i.e., the address, zip-code, the selling price and size (both in square and cubic feet)—as well as a large set of other features that may also impact the price of the house; see Appendix A of Gautier and Van Vuuren 2019). The two data sets are matched based on the registration code that is used for the land-lease contracts of our first data set.[27]

Statistics Amsterdam divides the city into 90 neighbourhoods, and we adopt their definition. Within a neighbourhood, the houses and economic status of the owners are approximately homogeneous. Our empirical implementation is therefore based on comparisons within neighbourhoods. Note that neighbourhoods in existence before 1896 have very few houses with land lease, while neighbourhoods developed after that year have very few houses with private land. This leaves us with 26 neighbourhoods that contain at least ten houses on private land and at least ten houses with a land-lease contract.

[26] The calculation of the prepayment is based on the net present value of the future land-lease payments. We therefore focus on the decision to buy a house that is paid x years in advance (instead of a house in private ownership) rather than on the decision to pay the land lease in advance.

[27] For this, we use a third data set of the municipality. See Gautier and Van Vuuren (2019) for details.

Table 2. Descriptive statistics of the data set.

	Private land	Landholds
Number of observations	22,348	5,684
Price	287,935	248,001
Size in square feet	940	878
Number of neighbourhoods	26	26
Neighbourhoods		
City centre	27.4	17.5
West	16.1	23.4
East	11.9	24.6
North	0.8	15.1
New-west	1.1	6.6
South-east	0.0	0.0
South	35.0	12.2

Table 2 provides some descriptive statistics of our final data set. We distinguish between private land (freeholds) and landholds. We find that the majority of the houses are freeholds, and that the prices of these houses are higher than the prices of houses with prepaid land lease. Note that the difference in prices can be partly explained by differences in size and location: houses with prepaid land lease are slightly smaller, and houses on private land are overrepresented in the more expensive areas in the city centre and in the southern parts close to that city centre. We correct for these differences by using neighbourhood dummies in our empirical analysis.

5.3. *Empirical implementation*

Before we can estimate the model with land-lease data, we must make additional assumptions to deal with the fact that houses are not identical. Let x_i be the observed individual characteristics of a house (including a full set of regional dummies). Assume that the price of a freehold can be written as

$$\log \widehat{p}_i = x_i \alpha + \upsilon_i, \tag{5.1}$$

where υ_i is the error term representing the unobserved characteristics of houses. We can estimate α in a first-stage regression using the freeholds.

Although $\widehat{p}_i$ is only observed for freeholds, we can still use the estimates of α to calculate $\widehat{p}$ for the leaseholds. We can then interpret $\widehat{p}$ as the price of an identical house that is a freehold. This can be substituted in the regression equation (4.1) of Section 4.1 and used to estimate the parameters on the left-hand side of that equation.

The price for an otherwise identical freehold is estimated consistently as long as the houses are representative (conditional on their observed characteristics). There are two reasons why we believe this is the case. First, we correct for many observable characteristics. That is, we estimate the regression equation (5.1) correcting for a full set of year dummy variables, neighbourhood, year-interacted-with-neighbourhood, type of house, size (in square, cubic feet and number of rooms), a categorical variable for the type of location of the house, maintenance outside and inside (9-point scale), whether the house is a monument and/or has a garage, balcony, attic, garden and/or roof terrace, type of heating, period of construction, type of insulation, number of bathrooms, whether the house is newly built, and whether the house is (partly) let and/or is used as a residential income property.[28] We find an R^2 value for this regression equation equal to 0.93. Second, we look only at neighbourhoods that were developed around the year of the introduction of land-lease rent. Hence, whether the house is a freehold or a leasehold depends on whether the block in which the house is situated was developed just before or just after the year 1896.

The final hurdle we must overcome in order to be able to use (4.1) is to determine future values of $L_{i,s}$. In practice, the city calculates the land-lease rents as a fixed fraction, γ_i, of the expected value of the house on private land. However, γ_i does vary between neighbourhoods. Hence, $\gamma_i = \gamma_{B(i)}$, where $B(i)$ is a function that maps individual houses into a particular neighbourhood. Based on these assumptions, we derive the following regression equation for houses for which the land-lease rents are not paid in advance:[29]

$$\log L_i - x_i \alpha = \log \gamma_{B(i)} + \upsilon_i, \tag{5.2}$$

where we can use the estimated value of α from the freeholds. Based on this, we can calculate the future land-lease payments for the houses that are paid in advance.

Before presenting the structural-form estimates of δ and β, we provide some reduced-form evidence that houses on private land are indeed more expensive. These results are discussed more extensively in a companion paper, Gautier and van Vuuren (2019). We find (using random effects) that freeholds are 13% more expensive than similar houses in the same neighbourhood with land lease. Moreover (using fixed effects), we find that for each additional year of prepayment, the house price increases by 0.2%. While this may be caused by present bias, it is also consistent with a geometric discount function and a positive interest rate. We thus require a model, like the one presented in Section 2, in order to establish whether we can interpret a relatively high market valuation of constant payments and a corresponding constant consumption pattern to be the result of present bias. This will be illustrated in the next section.

5.4. *Structural estimation of β and δ*

The results are presented in Table 3. We estimate four specifications: (1) a model with linear felicity; (2) a model with CARA felicity with $\beta = 1$; (3) a model with CARA felicity with $\beta \leq 1$; and (4) a model with CARA felicity with $\beta \leq 1$ and two groups of households. For the last specification, there is one agent type with $\beta \leq 1$ and another agent type with $\beta = 1$. Our first specification provides a precisely estimated value for δ, which is slightly less than 0.96. The second specification gives a similar value for δ, with only slightly higher standard errors. The third specification gives a value of 0.85 for β; we find, as in the simulations, that the standard error of β is larger than that of δ. The fourth specification allows for heterogeneity. In this specification, we find that 62% of the population has present bias, and that the estimate of β for this group is a bit lower than for the homogeneous case. However, the standard errors are large. This indicates that heterogeneity in the population not only requires stronger identification assumptions but also implies less precise estimates. Note also that allowing for present bias only leads to a small increase in R^2. This can be explained by the fact that the estimates of a are in general very small, implying an almost linear felicity

[28] Alternatively, one can use machine-learning tools such as Lasso or Ridge to correct for overfitting. However, this gives the same results since the tuning parameter is set to zero using cross-validation, and hence the same set of explanatory variables is included.

[29] Our working paper version, Gautier and Van Vuuren (2018), describes the details not only of how to deal with fixed and variable land lease but also of how best to take account of house price increases.

Table 3. Results of the structural-form model.

	(1)	(2)	(3)	(4)
δ	0.9591	0.9564	0.9566	0.9561
	(0.0003)	(0.0031)	(0.0033)	(0.0034)
β			0.8538	0.8026
			(0.0163)	(0.1521)
a ($\times$ 1000)		0.0276	0.0262	0.0311
		(0.0098)	(0.0138)	(0.0396)
χ				0.4152
				(0.9188)
Test against model (1)	–	6.53	7.02	1.69
	–	0.0059	0.0149	0.0021
Test against model (2)	–	–	80.4	3.39
	–	–	0.0000	0.0080
R^2	0.7196	0.7266	0.7326	0.7349

Notes: R^2 values are based on the regression equation (4.1), where we use the predicted values of $\widehat{p}_i$ from (5.1). Heteroskedasticity robust standard errors are in parentheses.

function. From the discussion of Section 3 we know that in that case, price data are not very responsive to the level of β. Finally, we compare the fit of our model with a simple reduced-form model in Appendix E.

6. FINAL REMARKS

This paper has shown that the identification of present bias in a Harris and Laibson (2001) type model using price data of contracts that specify payments at different moments in time requires strong assumptions. In the presence of heterogeneity in the short-term discount factor, the present bias parameters are identified only if agents of a given type buy different contracts. Simulations show that heterogeneity also reduces the precision of our estimates. Our empirical illustration uses land-lease contract data from Amsterdam. These contracts specify payments at different moments in time and are traded and priced in a competitive market. Under the assumption that all buyers are identical in terms of time preferences, we find the long-run discount factor (δ) to be 0.96 and the present bias parameter (β) to be 0.85: this corresponds to a long-term discount rate of 4.2% and a short-term rate of 22.5%. Our estimates may be sensitive to model misspecification and could benefit from the availability of micro-consumption and income data.

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SUPPORTING INFORMATION

Additional Supporting Information may be found in the online version of this article at the publisher's website:

Replication package

Co-editor John Rust handled this manuscript.

APPENDIX A: DERIVATION OF EQUATION (2.10)

The first-order condition of the consumer's problem follows from (2.9):

$$u'_C\left(c_{s,m}(\omega_{s,m})\right) \geq \beta_m \delta(1+r)V'_{s+1}(\omega_{s+1,m}, \mathcal{L}^{\sigma}_{s+1,m}), \tag{A.1}$$

with equality whenever $c_{s,m}(\omega_{s,m}) < \omega_{s,m} + y - L_s$ (i.e., when the credit constraints are not binding). For the moment, we make this assumption and look at the case in which the borrowing constraints are binding afterwards. Differentiating (2.8) with respect to $\omega_{s,m}$ and using (2.1) gives

$$W'_{s,m}(\omega_{s,m}, \mathcal{L}_s) = \beta_m \delta(1+r)V'_{s+1,m}(\omega_{s+1,m}, \mathcal{L}^{\sigma}_{s+1}) = u'_C\left(c_{s,m}(\omega_{s,m})\right), \tag{A.2}$$

where the last equality follows from the first-order condition (A.1). Subtracting (2.8) from β_m times (2.7) yields

$$W_{s,m}(\omega_{s,m}, \mathcal{L}^{\sigma}_{s,m}) - (1-\beta_m)u\left(c_{s,m}(\omega_{s,m})\right) = \beta_m V_{s,m}(\omega_{s,m}, \mathcal{L}^{\sigma}_{s,m}). \tag{A.3}$$

Taking first-order derivatives of (A.3) with respect to $\omega_{s,m}$ and substituting (A.2) in and then subsequently dividing by β_m yields

$$V'_s(\omega_{s,m}, \mathcal{L}_s) = u'_C\left(c_{s,m}(\omega_{s,m})\right)\left(c'_{s,m}(\omega_{s,m}) + \frac{1}{\beta_m}\left(1 - c'_{s,m}(\omega_{s,m})\right)\right). \tag{A.4}$$

Next, using the first-order condition (A.1) for $s-1$ (instead of s), using equality instead of inequality (because we assumed binding credit constraints) for this equation, substituting the result into (A.4) and taking logs gives

$$\begin{aligned} \log\left[u'_C(c_{s,m}(\omega_{s,m}))\right] &= \log\left[u'_C(c_{s-1,m}(\omega_{s-1,m}))\right] - \log\left((1+r)\beta_m\delta\right) \\ &\quad - \log\left(c'_{s,m}(\omega_{s,m}) + \frac{1}{\beta_m}\left(1 - c'_{s,m}(\omega_{s,m})\right)\right). \end{aligned} \tag{A.5}$$

Repeated backward substitution gives (2.9).

APPENDIX B: PROOF OF PROPOSITION 2.1

We start with the first part of the Proposition. Substitution of $\beta_m = 1$ into (A.4) and substitution of the result into the first-order condition (A.1) gives

$$u'_C(C_{s,m}) = \delta(1+r)u'_C(C_{s+1,m}).$$

Combining this equation for s and $s-1$, and using the definition of $\Delta(s)$ results in

$$\frac{u'_C(C_{s-1,m} - \Delta_m(s-1))}{u'_C(C_{s-1,m})} = \frac{u'_C(C_{s-1,m} - \Delta_m(s) - \Delta_m(s-1))}{u'_C(C_{s-1,m} - \Delta_m(s-1))}. \tag{B.1}$$

Note that $A(c) = \lim_{h\downarrow 0}(u'_C(c)/u'_C(c-h) - 1)$; hence, when $A(c-\Delta) > A(c)\ \forall\ \Delta > 0$, we have

$$\frac{u'_C(c-\varepsilon)}{u'_C(c)} > \frac{u'_C(c-\varepsilon-\Delta)}{u'_C(c-\Delta)}, \tag{B.2}$$

for every $\varepsilon > 0$ and $\Delta > 0$.[30] Therefore,

$$\frac{u'_C(C_{s-1,m} - \Delta_m(s-1))}{u'_C(C_{s-1,m})} > \frac{u'_C(C_{s-1,m} - \Delta_m(s) - \Delta_m(s-1))}{u'_C(C_{s-1,m} - \Delta_m(s))}, \tag{B.3}$$

[30] The proof of this statement is available upon request.

since both $\Delta_m(s)$ and $\Delta_m(s-1)$ are positive. Combining (B.1) and (B.3), and then taking account of the fact that the numerators are equal gives $u'_C(C_{s-1,m} - \Delta_m(s-1)) < u'_C(C_{s-1,m} - \Delta_m(s))$. Using the fact that u_C is concave implies that $u'_C(\cdot)$ is decreasing; after rewriting, this gives that $\Delta_m(s) > \Delta_m(s-1)$. The proof for the case that $A(c)$ is decreasing is similar. Now, we turn to the second part of the Proposition. If $\beta_m < 1$, then combining the first-order condition, taking into account that $c'_{s,m} > c'_{s-1,m}$ and using again the definition of $\Delta_m(s)$ yields

$$\frac{u'_C(C_{s-1,m} - \Delta_m(s-1))}{u'_C(C_{s-1,m})} < \frac{u'_C(C_{s-1,m} - \Delta_m(s) - \Delta_m(s-1))}{u'_C(C_{s-1,m} - \Delta_m(s-1))}. \tag{B.4}$$

For CARA, (B.2) holds with equality. Therefore

$$\frac{u'_C(C_{s-1,m} - \Delta_m(s-1))}{u'_C(C_{s-1,m})} = \frac{u'_C(C(s-1) - \Delta_m(s) - \Delta_m(s-1))}{u'_C(C_{s-1,m} - \Delta_m(s))}. \tag{B.5}$$

The result follows by substitution of (B.5) into (B.4) and the the fact that u is concave. The proof for the increasing-absolute risk aversion (IARA) case is trivial. □

APPENDIX C: PROOF OF PROPOSITION 3.1

Based on equations (A.1) and (A.4), we have that

$$\frac{u'_C(C_{s-1,m} - \Delta_m(s-1))}{u'_C(C_{s-1,m})} = \frac{u'_C(C_{s-1,m} - \Delta_m(s) - \Delta_m(s-1))}{u'_C(C_{s-1,m} - \Delta_m(s-1))}\Upsilon(\beta_m),$$

with

$$\Upsilon(\beta_m) \equiv \frac{1-(1-\beta_m)c'_{s+1,m}(\omega_{s+1,m})}{1-(1-\beta_m)c'_{s,m}(\omega_{s,m})},$$

which is a strictly increasing function (since $c'_{s+1,m} > c'_{s,m}$), with a maximumat $\beta_m = 1$. Using (B.5), we obtain that β_m solves the restriction

$$\Upsilon^* \equiv \Upsilon(\beta_m) = \frac{u'_C(C_{s-1,m} - \Delta_m(s-1))}{u'_C(C_{s-1,m} - \Delta_m(s))}$$

whenever $A(c)$ is constant. For the case in which $A(c)$ is decreasing we have likewise the restriction that

$$\Upsilon(\beta_m) = \frac{u'_C(C_{s-1,m} - \Delta_m(s-1))}{u'_C(C_{s-1,m} - \Delta_m(s))} = \Upsilon^*,$$

which immediately implies that the solution of β_m must be lower. □

APPENDIX D: PROOF OF THE FINAL STEP FOR IDENTIFICATION

Proof. Since agents are homogeneous, we drop subscript m. Instead of using the definition presented in the main text, we use here a slightly different definition for h,

$$\widetilde{h}(\sigma) = \frac{\partial P(H,\beta,\delta;\sigma,p_\sigma)}{\partial\beta}\Big/\frac{\partial P(H,\beta,\delta;\sigma,p_\sigma)}{\partial\delta}.$$

Note that in the case that we have homogeneity, as is assumed here, we obtain that

$$W_0(\omega_0,\mathcal{L}_0^\sigma) = W_0(\omega_0,\widehat{\mathcal{L}}_0^\sigma). \tag{D.1}$$

Total differentiation of (D.1) gives the following derivative of β:

$$\frac{\partial P(\beta,\delta;\sigma,p_\sigma)}{\partial\beta}=\frac{\partial(\widehat{W}_0-W_0)/\partial\beta}{\partial(\widehat{W}_0-W_0)/\partial\widehat{p}}. \tag{D.2}$$

A similar expression can be obtained for δ. Substituting (2.10) and (2.11) into (2.3), and then taking derivatives gives

$$\frac{\partial W_0}{\partial\beta}=\frac{W_0}{\beta}-u_C(y-rp_\sigma)+\beta\sum_{s=s^*}^{s^{**}}u'_C(\partial C_s(\omega_s))\delta^s\frac{C(\omega(s))}{\partial\beta}.$$

One can obtain a similar expression for $\partial\widehat{W}_0/\partial\beta$ by setting s^* and s^{**} equal to T. Substitution of these results into (D.2) makes the numerator of $\partial\widehat{p}/\partial\beta$ equal to

$$\frac{\partial\widehat{W}_0}{\partial\beta}-\frac{\partial W_0}{\partial\beta}=u_C(y-rp_\sigma)-u_C(y-r\widehat{p})-\beta\sum_{s=s^*}^{s^{**}}u'_C(C_s(\omega_s))\delta^s\frac{\partial C(\omega(s))}{\partial\beta}.$$

Similarly, we obtain from these equations that

$$\begin{aligned}\frac{\partial\widehat{W}_0}{\partial\delta}-\frac{\partial W_0}{\partial\delta}=&u_C(y-rp_\sigma)-u_C(y-r\widehat{p})-\beta\sum_{s=s^*}^{s^{**}}u'_C(C_s(\omega_s))\delta^s\frac{\partial C(\omega(s))}{\partial\delta}\\&+\beta\sum_{s=1}^{T}s(u_C(y-rp_\sigma)-u_C(c_s))\delta^{s-1}.\end{aligned}$$

Hence, we obtain that the division of the derivatives equals

$$\widetilde{h}(\sigma)=\frac{u_C(y-rp_\sigma)-u_C(y-r\widehat{p})-\beta\sum_{s=s^*}^{s^{**}}u'_C(C(s)\delta^s\frac{\partial C(s)}{\partial\beta}}{u_C(y-rp_\sigma)-u_C(y-r\widehat{p})-\beta\sum_{s=s^*}^{s^{**}}u'_C(C(s))\delta^s\frac{\partial C(s)}{\partial\delta}+\beta\sum_{s=1}^{T}s(u_C(y-rp_\sigma)-u_C(c_s))\delta^{s-1}}. \tag{D.3}$$

Note that the last term in the denominator does not appear in the numerator. This term also varies independently from s^* and s^{**}, which are functions of σ unless agents smooth their income from period 0 to T. The last term in the numerator depends only on how c_s changes with respect to σ, and for many levels of c_s it does not change for subsequent levels of σ (since consumption is at the credit constraints). This results in the fact that in general $\widetilde{h}(\sigma)$ is dependent on σ. The proof of identification is a direct result of the implicit function theorem. □

APPENDIX E: FIT OF THE STRUCTURAL-FORM MODEL RELATIVE TO A SIMPLE REDUCED-FORM REGRESSION

Consider the following regression equation:

$$\log(p_\sigma)-\log\widehat{p}_i=\Psi(\sigma_i)+\iota_i. \tag{E.1}$$

We can estimate this regression equation in two ways. First, we can use the virtual price of $\widehat{p}_i$ as calculated from α in (5.1). Second, we can calculate $\widehat{p}_i$ from the model, i.e., $\widehat{p}(p_{\sigma,i,\beta,\delta,a,\chi})$ in (4.1). Note that we do not fit this regression equation for our empirical implementation. In addition, (E.1) does not follow from our model since the relationship is more complex, depending not only on the relative price but also on the absolute value of the future land-lease payments. Rather, this reduced-form regression could be seen as an attempt to obtain a simple representation (as in Figure 3) using the heterogeneous sample of houses that we have in our data set. Figure E1 illustrates this relationship. The fit is best in the area with the most observations.

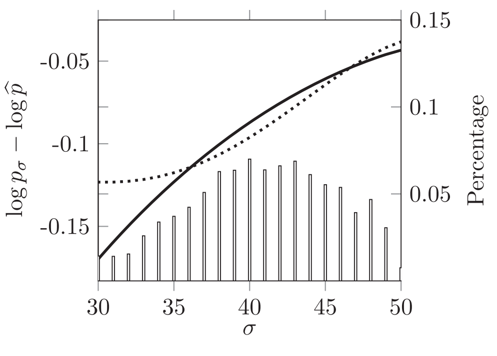

Figure E1. Fit of the structural-form model relative to a simple reduced-form regression.
Notes: the *y*-axis is the difference between the price for σ years of prepayment and the virtual price of a freehold. The dotted line plots the reduced-form regression, and the solid line plots the predicted values of the model. The bars represent the fraction of houses with contracts that are prepaid for σ years.